Bandwidth Estimation Using End-to-End Packet-Train Probing: Stochastic Foundation

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April 3, 2006
Outline

• Background
  – What is available bandwidth and why do we measure it?
  – Single-hop fluid model and existing techniques

• Stochastic Foundation
  – Single-Hop case
  – Multi-Hop case

• Experimental Verification

• Implications
Background: Basics

- **A network path**: a number of packet-forwarding hops

- **End points**: users of the path
  - Send packets to or receive packets from the path
Background: Basics

- A hop: a FIFO queue + a data transmission server

- Hop capacity $C$: transmission speed of the server in bits per second
- Cross-traffic rate $\lambda$: data arrived per time unit
Background: Definitions

- Available Bandwidth

Available bandwidth of hop 1

Utilized bandwidth of hop 1

Path available bandwidth is the minimum hop available bandwidth
Background: Definitions

- Tight Link

Non-tight link

Tight link

Non-tight links

A_1 \quad \lambda_2

A_2

A_3

\lambda_3

A_4

\lambda_4
Background: Motivation

• Why measure available bandwidth?
  – Useful to a lot of applications
    • TCP ramp-up
    • Server selection
    • Overlay topology optimization
    • ...

• Why measure from end points?
  – In the current Internet, end-users do not have access privileges of the data-forwarding hops
Background: Packet-Train Probing

- How to measure from endpoints?
  - Send probing packet-trains and infer bandwidth information from the input and output packet-train dispersions (i.e., the time gap between packets)
Background: More about Packet-Train

• Packet-train: a group of equally-sized packets

\[ n = 5 \]

• Packet-train parameters
  – Size \( s \) and length \( n \) (not changeable by the path)
Background: More about Packet-Trains

- **Packet-train**: a group of equally-sized packets

  \[ s = \frac{g}{r} \]

- **Packet-train parameters**
  - Size \( s \) and Length \( n \) (not changeable by the path)

- **Signals carried by a packet-train**
  - Dispersion \( g \) and rate \( r \) (changeable by the path)
Background: Single-Hop Fluid Model

• Most existing measurement techniques are designed based a single-hop fluid model

  cross-traffic rate $\lambda$

• Fluid cross-traffic
  – Infinitely small packet size
  – Constant arrival rate $\lambda$ at any time interval

• Path available bandwidth is $C-\lambda$ constantly
• Most existing measurement techniques are designed based on a single-hop fluid model.

**Background: Single-Hop Fluid Model**

- Input dispersion (or gap) $g_I$
- Output dispersion $g_O$
- Cross-traffic rate $\lambda$

• Output: a response of the path to the input
  - Input-output relation is called the response curve of the path.
Background: Single-Hop Fluid Curves

- Single-hop fluid gap response curve

\[ g_O = \begin{cases} \frac{g_I \lambda}{C} + \frac{s}{C} & g_I \leq \frac{s}{C - \lambda} \\ g_I & g_I \geq \frac{s}{C - \lambda} \end{cases} \]

\[ = \max \left( g_I, \frac{s + g_I \lambda}{C} \right). \]
Background: Single-Hop Fluid Curves

- Explanation using a Hop Workload Graph
  - Workload: the amount of data waiting for transmission in the queuing system, measured in transmission time
Background: Single-Hop Fluid Curves

- Explanation using a Hop Workload Graph
  - Workload: the amount of data waiting for transmission in the queuing system, measured in transmission time

\[ g_O = \frac{g_I \lambda}{C} + \frac{s}{C} \]
Background: Single-Hop Fluid Curves

- Single-hop fluid rate response curves:

\[ r_O = \min \left( r_I, \frac{r_I}{r_I + \lambda} C \right). \]

\[ \frac{r_I}{r_O} = \max \left( 1, \frac{r_I + \lambda}{C} \right). \]
**Background: Single-Hop Fluid Curves**

- Existing techniques are based on the single-hop fluid curves

PTR searches for the turning point.

- TOPP measures the second linear segment and applies linear regression to compute $C$ and $\lambda$

- Spruce uses this point, assuming $C$ is known
Background: Limitations

• Major limitations of single-hop fluid models
  – Gives no proof that the model also applies to bursty cross-traffic
  – Ignores the impact of non-tight links
  – Provides no insights on the impact of packet train parameters on measurement accuracy

• Existing techniques were observed to produce 100% errors without knowing why

• A stochastic foundation is needed
  – To address these issues
  – To better understand the sources of measurement errors in the current techniques
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• Implications and Future Work
Single-Hop Case: Goal

- Derive the single-hop response curve using a packet-level bursty cross-traffic arrival
Single-Hop Case: Adapting Fluid Models

- Previous use of fluid models in bursty cross-traffic

\[ E[g_O] = \begin{cases} \frac{s+g_I \lambda}{C} & g_I \leq \frac{s}{C-\lambda} \\ g_I & g_I \geq \frac{s}{C-\lambda} \end{cases} \]

- In bursty cross-traffic, \( g_O \) varies, take the statistical average of \( g_O \) as the output dispersion

- In bursty cross-traffic, traffic arrival rate also varies, interpret \( \lambda \) as the long-term average arrival rate

- However, even with this adaptation, we show that the fluid model is not valid in general
Single-Hop Case: Stochastic Curve

- Stochastic Response Curve
  - We present the following gap response curve in bursty cross-traffic:

\[
E[g_O] = \frac{g_I \lambda + s}{C} + \frac{E[I]}{n - 1}
\]

\[
= g_I + \frac{E[R]}{n - 1}.
\]

- The two additional terms do not show up in fluid traffic, but do have an effect in bursty cross-traffic.
Single-Hop Case: What is $E[I]$?

- $I$ is the random variable indicating the hop idle time during the packet-train arriving interval.
Single-Hop Case: What is $E[R]$?

- $R$ is the extra queuing delay imposed on the last packet by the preceding packets in the same probing train.
Single-Hop Case: Intuitive Interpretation

• The two expressions describe $E[g_O]$ from two different angles

\[ E[g_O] = \frac{g_I \lambda + s}{C} + \frac{E[I]}{n - 1} \]

\[ = g_I + \frac{E[R]}{n - 1} \]

Hop activities between the departures of the pair

What causes the difference between the input and output dispersion?
Single-Hop Case: Response Deviation 1

• The two additional terms cause the stochastic response curve to deviate from the fluid curve

\[
E[g_O] - \max (g_I, \frac{g_I \lambda + s}{C}) = \begin{cases} 
\frac{E[I]}{n - 1} & r_I \geq C - \lambda \\
\frac{E[R]}{n - 1} & r_I \leq C - \lambda
\end{cases}
\]

Response Deviation \( \beta \)
Single-Hop Case: Response Deviation 2

- Response Deviation as a Function of $g_I$

When $g_I > s/(C-\lambda)$, $\beta$ monotonically decreases and asymptotically converges to 0.
Single-Hop Case: Response Deviation 3

- Transformed rate response curve

\[ E[r_I/r_O] \]

\[ r_{IC} \]

\[ \lambda \]

\[ C \]

\[ C - \lambda \]

\[ \alpha \]

Fluid Response Curve

Caused by \( E[I] \)

Caused by \( E[R] \)
Single-Hop Case: Packet-Train Parameters

- When packet train length $n$ increases, response deviation vanishes.
- Consider a packet-train of infinite length
  - When $r_I < C - \lambda$, queue is stable, mean queuing delay is bounded, so is the extra queuing delay term
    \[ E[g_O] = g_I + \frac{E[R]}{n - 1} \]
  - When $r_I > C - \lambda$, queue goes unbounded, the amount of hop idle time is bounded
    \[ E[g_O] = \frac{g_I \lambda + s}{C} + \frac{E[I]}{n - 1} \]
Single-Hop Case: Summary

- Two additional terms show up in bursty cross-traffic, causing the single-hop stochastic curve to deviate from the fluid curve.

- As packet-train length $n$ increases, the stochastic curve approaches the fluid curve.

- **Conclusion:** The fluid curve is a valid first-order approximation of the stochastic curve only when packet-train length is sufficiently large.
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Multi-Hop Case: Problem Statement

- An $N$-hop path probed by packet trains of length $n$

- **Goal**: understand the relationship between $E[G_N]$ and $G_0$ under arbitrary cross-traffic
  - That is the multi-hop probing response curve
Multi-Hop Case: Simplest Settings

• Consider fluid cross-traffic with one-hop persistent routing

• Mathematically:

\[ g_i = \max \left( g_{i-1}, \frac{s + \lambda_i g_{i-1}}{C_i} \right) \]
Multi-Hop Case: Relaxing Fluid Constraint

• When relaxing the fluid constraint, we get

\[
E[G_i] = \frac{E[G_{i-1}]\lambda_i + s}{C_{i-1}} + \frac{E[I_i]}{n - 1}
\]

\[
= E[G_{i-1}] + \frac{E[R_i]}{n - 1}.
\]

• The response deviation at link \(i\) is

\[
\beta_i = E[G_i] - g_i = \begin{cases} 
\beta_{i-1} + \frac{E[R_i]}{n-1} & g_i = g_{i-1} \\
\frac{\beta_{i-1}\lambda_i}{C_i} + \frac{E[I_i]}{n-1} & g_i > g_{i-1}
\end{cases}
\]
Multi-Hop Case: Effect of Packet-Train Length

• In one-hop persistent routing, it is easy to show using induction that

$$\beta_i \to 0, \text{ as } n \to \infty$$

• Conclusion: Multi-Hop curve approaches from above its fluid counterpart as packet-train length increases
  – This result also applies to arbitrary CT routing
  – Similar reasons, complex math description
Multi-Hop Case: Fluid Response Curves

- Multi-Hop Fluid Curves for Different CT Routing
Multi-Hop Case: Stochastic Response Curves

\[ E[r_I/r_O] \]

Non-elastic deviation, stays constant

Elastic deviation, diminishes when train length increases

Multi-hop stochastic

Multi-hop fluid

Single-hop fluid

1

\( r_I \)

\( C - \lambda \)

\( A_2 \)
Multi-Hop Case: Stochastic Response Curves

\[ E[r_I/r_O] \]

- Elastic deviation, diminishes when train length increases
- Non-elastic deviation, stays constant

- Multi-hop stochastic
- Multi-hop fluid
- Single-hop fluid
Multi-Hop Case: Stochastic Response Curves

\[ E[\frac{r_I}{r_O}] \]

Elastic deviation, diminishes when train length increases

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\( r_I \)

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Multi-Hop Case: Stochastic Response Curves

\[ E[\frac{r_I}{r_O}] \]

- Elastic deviation, diminishes when train length increases
- Non-elastic deviation, stays constant

- Multi-hop stochastic
- Multi-hop fluid
- Single-hop fluid

1

\[ C' \lambda \] \[ A_2 \]

\[ r_I \]
Multi-Hop Case: Stochastic Response Curves

\[ E[r_I/r_O] \]

- Non-elastic deviation, stays constant
- Elastic deviation, diminishes when train length increases

- Multi-hop stochastic
- Multi-hop fluid
- Single-hop fluid

1

C - \lambda

A_2
Multi-Hop Case: Stochastic Response Curves

\[ E[r_I/r_O] \]

Non-elastic deviation, stays constant

Elastic deviation, diminishes when train length increases

Multi-hop stochastic

Multi-hop fluid

Single-hop fluid

1

\( C-\lambda \)

\( A_2 \)

\( r_I \)
Multi-Hop Case: Stochastic Response Curves

\[ E\left[ \frac{r_I}{r_O} \right] \]

Non-elastic deviation, stays constant

Elastic deviation, diminishes when train length increases

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1

\[ C - \lambda \]

\[ A_2 \]
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Experimental Verification: Roadmap

• Single-Hop Response Curves
  – NS2 Simulation in Poisson traffic

• Multi-Hop Response Curves
  – Emulab testbed experiment with real traffic traces
  – Real Internet measurement over the RON testbed
Experimental Verification: Single-Hop

- Single-hop path 10mb/s capacity with 3mb/s Poisson cross-traffic
Experimental Verification: Emulab 1

- Emulab Testbed Settings
Experimental Verification: Emulab 2

• One-Hop Persistent Routing Case
Experimental Verification: Emulab 3

- Path Persistent Routing Case

![Graph showing path persistent routing case with multiple lines representing different values of n, and labels for single-hop and multi-hop fluid.]
Experimental Verification: Real Internet Data

- We measure the rate response curves for 272 Internet paths over the RON testbed.

- Parameters:
  - Input rates: from 10 to 150 mb/s with step 5 mb/s
  - Packet-train length: 129 packets
  - Packet-size: 1500 bytes
  - For each rate, we use 200 trains to estimate $E[G_N]$

- Experiment durations are 20-100 minutes.
Experimental Verification: Internet Data

• Cornell → CMU, 5/25/2005
Experimental Verification: Internet Data

- Ana1-gblx → Cornell, 4/29/2005
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Implications: TOPP

- TOPP uses packet-pairs to measure the stochastic response curve and implicitly assumes that it is the same as the fluid curve
  - Our results show that the two are not the same even for a single-hop path (response deviation)
  - Increasing the packet-train length can reduce the response deviation to a negligible level, and make TOPP work in practice
  - Our Internet measurement shows a length of several tens (e.g., 30) is usually enough
Implications: TOPP

- Use the Poisson simulation as an example

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<thead>
<tr>
<th>C</th>
<th>λ</th>
<th>A</th>
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<tr>
<td>10</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>35.81</td>
<td>32.38</td>
<td>3.43</td>
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</table>
Implications: Spruce 1

- Spruce is unbiased in a single-hop path

At this input rate point, the two response curves agree with each other.
Implications: Spruce 2

- Measurement biases of Spruce in multi-hop paths.
**Implications: Spruce 3**

- Spruce measurement biases

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Elastic Bias</th>
<th>Non-Elastic Bias</th>
<th>Total bias</th>
<th>Real avail-bw</th>
<th>Spruce Measurement</th>
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<tbody>
<tr>
<td>Emulab-1</td>
<td>53.76</td>
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<td>Emulab-2</td>
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<td>0</td>
<td>38</td>
<td>96</td>
<td>58</td>
</tr>
</tbody>
</table>
Implications: PTR and pathload

• Pathload and PTR are related to searching for the turning point in the single-hop fluid response curve
  – Since they are using long trains, they are often immune to measurement bias, even in a multi-hop path.
  – Recall that using long trains, the multi-hop stochastic curve will approach the single-hop fluid curve within that region.
Wrap-up

• Conclusion
  – We derived the stochastic probing response curves in both single-hop and multi-hop paths
  
  – Our results provide a stochastic justification of the existing techniques using long-trains
  
  – Also uncover the sources of measurement biases for the techniques using short trains and possible ways to overcome the biases
  
  – Lead to new approaches for measuring the tight link capacity
Wrap-up

- Several Related Papers
  
  
  